Electric field induced narrowing of exciton line width.

I. V. Ponomarev,* L. I. Deych, and A. A. Lisyansky
Department of Physics,
Queens College of the City University of New York,
Flushing, NY 11367
(Dated: February 2, 2008)

Considering effects of electric field on the low temperature absorption line of quantum well excitons, we show that, for moderate strength of the electric field, the main contribution to the field dependence of the line-width results from field induced modifications of inhomogeneous broadening of excitons. We find that the strength of the random potential acting on quantum well excitons due to alloy disorder and interface roughness can either decrease or increase with field depending upon the thickness of the well. This means that under certain conditions one can observe counterintuitive narrowing of exciton spectral lines in electric field.

PACS numbers: 71.35.Cc,73.21.Fg, 78.67.De.

In the case of three-dimensional excitons, it is well known that application of an electric field significantly reduces the exciton life-time because of a finite probability of exciton tunnelling through the field distorted potential barrier. An obvious spectroscopic consequence of this effect is significant broadening of exciton spectral lines at moderate electric fields. However, excitons confined in a quantum well (QW) are much more robust with respect to the electric field, $\mathcal{E}$, applied perpendicular to the plane of a QW. As a result, QW excitons demonstrate an appreciable field induced shift of their spectral lines, while the spectral width does not change too much. The physical origin of this quantum confined Stark effect [1] (QCSE) lies in the significantly increased stability of QW excitons compared to the bulk case. Indeed, Stark broadening shows strong exponential dependence on field:

$$\Gamma_{Stark} \sim E_0 \exp\left(-2\hbar^2/3m|e|\mathcal{E}\ell^3\right), \tag{1}$$

where E_0 is a typical value of the resonance energy, m is an effective particle mass, and ℓ is a tunneling length. In QWs, ℓ is determined by the confining potential of the well rather than by the binding energy of excitons. Since the former is one or two orders of magnitude greater than the latter, it is clear that QW excitons in the perpendicular field can withstand much stronger fields than their three dimensional counterparts. QCSE has received a great deal of attention during the last two decades, and was exploited in a number of electro-optic devices. However, the issue of electric field induced changes in the exciton line width, and their origin, still remains largely unstudied (see Ref. 2).

In this paper we consider electric field induced modification of exciton inhomogeneous broadening, which determines exciton line width at low temperatures[3, 4, 5]. We show that modification of the random potential caused by a reconstruction of electron-hole wave functions in the applied electric field results in a *power law* field dependence of exciton line width, which yields much stronger change in the line width than the exponential dependence caused by Stark effect, Eq. (1).

We find that for QWs whose thickness, L, is smaller than some critical value, L_{cr}, an electric field actually reduces fluctuations of this potential resulting in a counterintuitive *narrowing* of the exciton line width with the electric field. When $L > L_{cr}$, the sign of the electric field contribution to the line width changes and exciton lines becomes broader with field increase. The critical thicknesses, as well as L and $\mathcal{E}$ dependencies of the exciton line width, are different for compositional disorder and interface roughness mechanisms of the inhomogeneous broadening. Therefore, experimental observation of field induced changes in the low temperature exciton line width can yield a unique method of characterization of QWs allowing separation of these two contributions to the exciton's spectral broadening.

Let us consider QW formed by a binary semiconductor, AB, as a barrier material, and a ternary disordered alloy, $AB_{1-x}C_x$, as a well. Throughout the paper we use effective atomic units: the effective Bohr radius for length, $a_B = \hbar^2\epsilon/\mu^* e^2$, $E_B = \mu^* e^4/\hbar^2\epsilon^2 \equiv 2$ Ry for energy, and reduced electron-hole mass μ^* for masses, $1/\mu^* = 1/m_e^* + 1/m_h^*$. In the isotropic effective mass approximation, the Hamiltonian for the exciton in QW with disorder is

$$H = H_0^e(\mathbf{r}_e) + H_0^h(\mathbf{r}_h) - |\mathbf{r}_e - \mathbf{r}_h|^{-1} + U^e(\mathbf{r}_e) + U^h(\mathbf{r}_h), \tag{2}$$

where $U^{e(h)}(\mathbf{r}_{e(h)})$ are disorder induced potentials, and $H_0^{e(h)}(\mathbf{r}_{e(h)})$ are Hamiltonians for an electron(hole) in QW:

$$H_0^{e(h)}(\mathbf{r}) = \frac{p_{e(h)}^2}{2m_{e(h)}} + V_0^{e(h)}\theta\left(z^2 - L^2/4\right) \mp \frac{F^{e(h)}z}{2m_{e(h)}}, \tag{3}$$

where $F^{e(h)} = 2m_{e(h)}|e|\mathcal{E}$, and $\theta(z)$ is the step function. Inhomogeneous broadening of excitons in such a well is determined by a combination of two types of disorder:

$$U^{e(h)}(\mathbf{r}_{e(h)}) = U^{e(h)}_{alloy}(\mathbf{r}_{e(h)}) + U^{e(h)}_{int}(\mathbf{r}_{e(h)}). \tag{4}$$

Compositional disorder, $U^{e(h)}_{alloy}$, arising due to concentration fluctuations in a ternary component, which produce local band gap fluctuations [6, 7, 8], and interface roughness, $U^{(e,h)}_{int}$, caused by the formation of monolayer islands on the QW interfaces that results in local changes in the QW thickness [9, 10, 11, 12].

Usually, the exciton binding energy in QW is much larger than disorder-induced local energy fluctuations. Therefore excitons are expected to move through QW as a whole entity. It is formalized in a representation of the total wave function of the electron-hole pair in the form of a product,

$$\Psi(\mathbf{r_e}, \mathbf{r_h}) = \Phi(\mathbf{R})\psi(\boldsymbol{\rho})\chi_e(z_e)\chi_h(z_h), \tag{5}$$

where $\mathbf{r}_{h,e} = (\boldsymbol{\rho}_{h,e}; z_{h,e})$, $\boldsymbol{\rho} = \boldsymbol{\rho}_e - \boldsymbol{\rho}_h$, $\mathbf{R} = (m_e\boldsymbol{\rho}_e + m_h\boldsymbol{\rho}_h)/M$, $\Phi(\mathbf{R})$ is a wave function for center-of-mass lateral motion, $\psi(\boldsymbol{\rho})$ is an exciton relative lateral motion wave function, and $\chi_{e,h}(z_{e,h})$ are one-dimensional electron and hole QW ground state wave functions. Functions ψ and χ are solutions of the corresponding Schrödinger equation for a perfect QW without disorder, while the Schrödinger equation for the center-of-mass motion includes effective random potentials, $V_{eff}(\mathbf{R}) = V_{int}(\mathbf{R}) + V_{alloy}(\mathbf{R})$, obtained from the averaging of the original random potentials $U^e(\mathbf{r_e})$, $U^h(\mathbf{r_h})$ over $\boldsymbol{\rho}$ and z coordinates. (In these calculations we do not take into account disorder-induced renormalization of functions ψ, $\chi_{e,h}$ and the corresponding energies [3], which results in effective decreasing of Bohr's radius λ. This effect does not change qualitative conclusions of our work [see Eqs. (18) and (19)].)

Our primary goal is to calculate the variance of the effective random exciton potential defined as

$$W = \sqrt{\langle V_{eff}(\mathbf{R})^2\rangle}. \tag{6}$$

This parameter determines a number of experimentally observable quantities such as the exciton radiative life-time and the absorption line width [3, 4, 5]. The radiative life-time can be extracted from the exciton absorption line-shape [8], whose calculation in the dipole approximation is equivalent to the estimation of the optical density function: $A(\varepsilon) = \left\langle \sum_i \left| \int d^2\, R\Phi_i(\mathbf{R}) \right|^2 \delta(\varepsilon - \varepsilon_i) \right\rangle$. The interpolation procedure [3, 8] gives an asymmetric line shape towards high frequencies. In many cases however, for estimation of the line-width it is reasonable [4, 5] to consider that the underlying disorders are described by the Gaussian random processes. Then the shape of the exciton line is also Gaussian, and the corresponding full-width-at-half-maximum is given by $\Delta = \sqrt{8\ln(2)}W$.

Since contributions from the alloy and the interface disorders can be considered statistically independent of each other, $W^2_{tot} = W^2_{alloy} + W^2_{int}$. Estimations show that usually both disorders yield comparable contributions to the total width. This imposes an additional difficulty for experimental identification of the interface quality in QWs from optical spectra, since absolute values of each contribution are usually unknown.

The effective random potentials acting on the exciton's center-of-mass, for each type of disorder, is presented as a sum of two terms $V(\mathbf{R}) = V_h(\mathbf{R}) + V_e(\mathbf{R})$, representing hole and electron contributions,

$$V_{h,e} = \int U_{h,e}(\mathbf{R} \pm m_{e,h}\boldsymbol{\rho}/M; z)\psi^2(\boldsymbol{\rho})\chi^2_{h,e}(z)\, d^2\rho dz. \tag{7}$$

Correspondingly, each variance will have three terms $W^2 = \langle V_h^2 + 2V_hV_e + V_e^2\rangle$. For a QW with a heavy hole and light electron ($m_h \gg m_e$) as in $In_xGa_{1-x}As/GaAs$ heterostructures, the main contribution stems from the hole-hole part due to the enhancement factor (M/m_e) which appears after averaging over lateral coordinates $\boldsymbol{\rho}$ in Eq. (7). Considering only this case, we neglect terms containing a V_e factor. Then the microscopic potential representing the alloy can be presented as (hereafter the subscript "h" is omitted) [8]

$$U_{alloy}(\mathbf{r}) = \alpha\xi(\mathbf{r})\theta\left(L^2/4 - z^2\right)/N, \tag{8}$$

where $\theta(z)$ is a step-function, N is the concentration of lattice sites ($N = 4/a^3_{lat}$ for zincblende materials, a_{lat} is a lattice constant), $\xi(\mathbf{r})$ is the random fluctuation of the local concentration of atoms in the alloy from the average value xN, and $\alpha = dE_v/dx$ characterizes the rate of shift of the valence bands with composition x. The interface roughness potential can be presented in the following form [12, 13]

$$U_{int}(\mathbf{r}) = V_0\left[\eta_1(\boldsymbol{\rho})\delta(z + L/2) - \eta_2(\boldsymbol{\rho})\delta(z - L/2)\right], \tag{9}$$

where $\delta(z)$ is a δ-function, V_0 is a hole off-set band energy. Random functions $\eta_{1,2}(\boldsymbol{\rho})$ with zero mean characterize a deviation of the ith interface from its average position.

The statistical properties of alloy and interfacial roughness are characterized by the correlators [6, 12, 13, 14, 15]:

$$\langle \xi(\boldsymbol{r}_1)\xi(\boldsymbol{r}_2)\rangle = x(1-x)N\delta(|\boldsymbol{r}_1-\boldsymbol{r}_2|), \quad (10)$$

$$\langle \eta_i(\boldsymbol{\rho}_1)\eta_j(\boldsymbol{\rho}_2)\rangle = h^2 f_{ij}\zeta(|\boldsymbol{\rho}_1-\boldsymbol{\rho}_2|), \quad (11)$$

where h is an average height of interface inhomogeneity, and $\langle\cdots\rangle$ denotes an ensemble average. For the interface height-height correlator we assume that the dependence of both diagonal and non-diagonal correlations on the lateral coordinates $\boldsymbol{\rho}$ is described by the same function $\zeta(\boldsymbol{\rho})$. The diagonal elements f_{ii} are different if two interfaces are grown under different conditions, which happens naturally for *GaAs* based structures. (Growth of a ternary alloy on *GaAs* occurs differently from growth of *GaAs* on the alloy; besides using techniques of growth interruption one can significantly modify statistical properties of the grown interfaces.) The non-diagonal element $f_{12}(L/\sigma_\parallel)$ introduces correlations between different interfaces. The respective quantity, which can be called the *cross-* or *vertical-correlation function* [13, 14, 15, 16], is a function of the average width of the well and is characterized by the vertical correlation length $\sigma_\parallel$. The presence of these correlations suppresses the interface disorder contribution into inhomogeneous broadening, especially, for narrow QWs [13, 17].

In order to calculate the effective potential, one needs to know the exciton wave functions ψ and χ for an ideal QW in the perpendicular uniform electric field. They can be found with the help of the variational method. It is a well-known fact [1, 18] that lateral relative motion is very weakly affected by perpendicular electric field. The corresponding trial function can be chosen in a form of the hydrogen $1S$-like orbital, $\sqrt{2/\pi\lambda^2}\exp(-r/\lambda)$, with the quasi-two-dimensional Bohr's radius λ as a variational parameter. In principle, the one-dimensional single-particle function $\chi(z)$ for a hole in QW, which satisfies the Schrödinger equation

$$\left[p_z^2/2m + V_0\theta\left(z^2 - L^2/4\right) + Fz/2m\right]\chi = E\chi, \quad (12)$$

can be found exactly in terms of the Airy functions. This solution corresponds to a quasi-stationary hole state, and describes a possibility for the hole to tunnel out of the well. It diverges at infinity, and is not suitable, therefore, for calculation of the effective potential, Eq. (7). In this paper, however, we are interested in the range of parameters where Stark broadening is small. This restricts our consideration to a certain region on the F-L plane (grey shaded area in Fig. 1), where the exponent of Eq. (1) is smaller than unity. It is worth to note, that these fields can reach very high values since the relevant energy scale is the confining QW potential rather than the Coulomb interaction between the hole and the electron. For example, for $In_{0.18}Ga_{0.82}As/GaAs$ QWs the maximum in Fig. 1 corresponds to the field 8×10^5 V/cm.

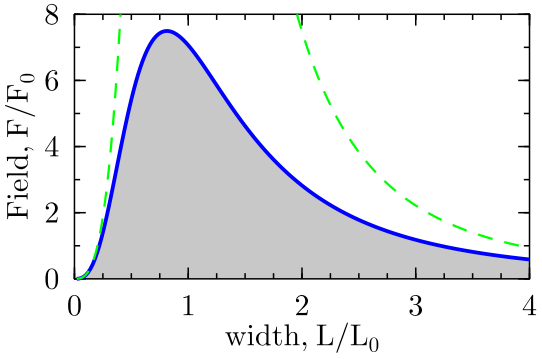

FIG. 1: (Color online) Phase diagram F-L of dominant contributions to the exciton line-width: the gray shaded region - disorder induced mechanisms, the outside region - Stark broadening. Dashed lines are the shallow well and the infinite well approximations. The QW width is measured in units $L_0 = \pi/u$, which determines the number of levels in the QW, and F_0 defines the natural scale for electric field (see text).

Previous studies of QCSE showed [19] that within this range of parameters, an approximation of $\chi(z)$ by a real function, which can be found with the help of variational method, gives a very good description of both the energy and the wave function of a hole in the presence of a perpendicular electric field. Reasonable results can be obtained

even for the simplest one-parameter variational function of the following form [18]:

$$\chi(z;\beta) = B(\beta,k)\exp(-\beta z)\chi_0(z), \tag{13}$$

where the wave function, $\chi_0(z)$, represent the hole ground state in QW without the electric field:

$$\chi_0(z) = \begin{cases} \cos(kz), & z \le |L/2| \\ \exp\left[-\kappa(|z| - L/2)\right], & z \ge |L/2|, \end{cases} \tag{14}$$

B is the normalization constant:

$$B(\beta,k) = \sqrt{\frac{2\beta(\kappa^2-\beta^2)(k^2+\beta^2)}{k^2\left[2\kappa\beta\cosh(\beta L) + (\kappa^2+\beta^2)\sinh(\beta L)\right]}}, \tag{15}$$

and we introduced the following notations $k = \sqrt{2mE}$, $u = \sqrt{2mV_0}$, and $\kappa = \sqrt{u^2-k^2}$. The wave number k is given by a root of the transcendental equation, $kL/\pi = 1 - (2/\pi)\arcsin(k/u)$. Eq. (14) for $\chi_0(z)$ guarantees a continuity of the wave function and its derivative at interfaces $z = \pm L/2$. Parameter u defines a natural length scale for the QW width, $L_0 = \pi/u$. It counts the number of levels in QW: $[L/L_0] = n+1$.

In general, the solution for the variational parameter $\beta(F,L,u)$ can be found only numerically. One can show, however, that for a moderate field, β is proportional to the electric field, $\beta = C(L)F$, where

$$C(L) = \frac{1}{\kappa^2} - \frac{1}{k^2} + \frac{L}{\kappa}\frac{1 + L\kappa + L^2\kappa^2/6}{2 + L\kappa}. \tag{16}$$

The constant $C(L)$ introduces the natural scale for electric field units $F_0 = C(L)^{-3/2} \equiv \ell^{-3}$, where ℓ defines an average extension of wave function in QW. Analytical expression for $C(L)$ can be obtained in two important limits: a very wide well, which can be approximated by an effective infinite QW [19] with $k \approx \pi/L$, $\kappa \approx u$, and a very narrow shallow QW [18], which can be described by a model of a δ-functional QW potential with $k \approx u$, $\kappa \approx mV_0L$:

$$C_\infty = \frac{L^2}{2}\frac{\pi^2-6}{6\pi^2},\ C_\delta = \frac{1}{2(mV_0L)^2}. \tag{17}$$

Using correlators, Eqs. (10) and (11), and the wave functions $\psi(\rho)$ and $\chi(z;\beta)$ we obtain the following expressions for alloy and interface roughness variances:

$$W^2_{alloy} = \frac{a^3_{lat}x(1-x)}{8\pi\lambda^2}\frac{\alpha_h^2M^2}{m_e^2}\int_{-L/2}^{L/2}\chi(z;\beta)^4\,dz, \tag{18}$$

$$W^2_{int} = V_0^2h^2G(y)\left[f_{11}\chi_L^4 + f_{22}\chi_R^4 - 2f_{12}\chi_L^2\chi_R^2\right], \tag{19}$$

where $\chi_{L,R} \equiv \chi(\mp L/2;\beta)$. The function $G(y)$, defined as [13]

$$G(y) = \int d^2\rho d^2\rho' \psi^2(\rho)\psi^2(\rho')\zeta\left(|\boldsymbol{\rho}' - \boldsymbol{\rho}|m_e/M\right), \tag{20}$$

with $y = \sqrt{2}\sigma_\perp M/(\lambda m_e)$, depends on the lateral correlations of interface roughnesses, characterized by the in-plane correlation radius $\sigma_\perp$.

Since, for small F, parameter β is proportional to the field, both Eqs. (18) and (19) can be expanded in terms of F:

$$W^2_{all}(F) \approx \Omega_{all}\left[\gamma_0^{(all)} + \gamma_2^{(all)}F^2\right], \tag{21}$$

$$W^2_{int}(F) \approx \Omega_{int}\left[\gamma_0^{(int)} + \gamma_1^{(int)}F + \gamma_2^{(int)}F^2\right], \tag{22}$$

where

$$\Omega_{all} = \left[\kappa^2a^3_{lat}x(1-x)\alpha_h^2M^2\right] / \left[(\kappa L+2)^2 2\pi\lambda^2m_e^2\right], \tag{23}$$

$$\Omega_{int} = 4\kappa^2k^4V_0^2h^2G(y)/[(\kappa L+2)^2u^4], \tag{24}$$

$$\gamma_0^{(int)} = f_{11} + f_{22} - 2f_{12}(L), \tag{25}$$

and all other γ_i are also monotonic functions of the QW width. Eqs. (21) and (22), which present the main results of the paper, show that in the range of parameters where the Stark width is exponentially small, there exists a strong *power law* field dependence of inhomogeneous exciton broadening caused by the field induced changes in the variance of the effective exciton potential.

The first remarkable feature of Eqs. (21) and (22) that we would like to point out is the presence of thw linear-in-field term (see Fig. 2) in the interface roughness

$$\gamma_1^{(int)}(L) = 2LC(L)\,(f_{11} - f_{22})\,. \tag{26}$$

One can see that this term results from asymmetry between two interfaces of the well, which manifests itself through different roughnesses, $f_{11} \neq f_{22}$. In $GaAs$ based heterostructures, this asymmetry appears naturally because of the polar nature of $GaAs$, but it can also be engineered by preparing different interfaces under different growth conditions. The presence of the linear term gives rise to an interesting effect: one can switch between field induced narrowing or broadening of the exciton line by simply changing the polarity of the applied field. This effect has a simple physical interpretation: if QW holes are pushed by the field toward a less disordered interface, the exciton line narrows, but it broadens in the opposite situation.

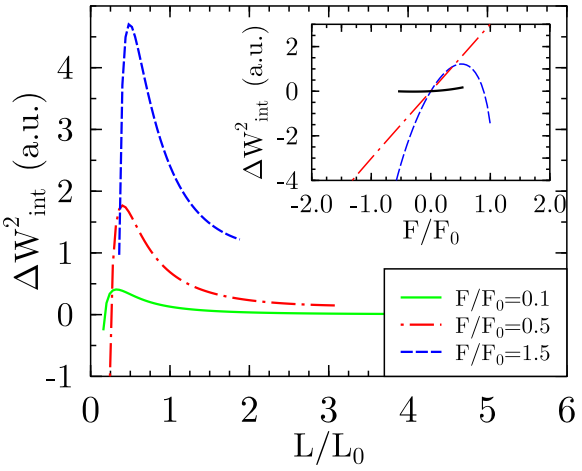

FIG. 2: (Color online) The field dependent part of the variance, $\Delta W^2_{int} = \Omega\left[\gamma_1 F + \gamma_2 F^2\right]$, as a function of the QW width for the interface roughness contribution with different corrugation at interfaces ($f_{11} = 4, f_{22} = 1$) for three values of the electric field: $F/F_0 = 0.1, 0.5, 1.5$. Curves are drawn only for the gray shaded area of Fig. 1, where disorder contributions to the exciton line width dominate. Inset: ΔW^2_{int} as a function of electric field for three QW widths: $L/L_0 = 0.3$ (dashed), $L/L_0 = 0.55$ (dotted-dashed), and $L/L_0 = 3$ (solid). Note that $L/L_0 = 0.55$ corresponds to the case, when the second order in field term disappears (see text).

Quadratic in the field terms in Eqs. (21) and (22) also possess nontrivial properties (see Fig. 3). In the limit of shallow δ-functional QWs, factors $\gamma_2(L)$ can be presented as

$$\gamma_2^{(all)} = -2C_\delta^2\left[(mV_0L)^{-2} - L^2/3\right], \tag{27}$$

$$\gamma_2^{(int)} = -2C_\delta^2\left[\frac{\gamma_0^{(int)}}{(mV_0L)^2} - 2L^2\left(\gamma_0^{(int)} + 4f_{12}\right)\right]. \tag{28}$$

At small widths, $L < L_{cr}$, the first term in square brackets in both equations dominates making respective contributions to the line width negative. In the opposite limit of an effective infinite wall, these factors are positive:

$$\gamma_2^{(all)} = C_\infty^2 L^2\left[1/3 - 3/\pi^2\right] \tag{29}$$

$$\gamma_2^{(int)} = C_\infty^2\left[\gamma_0^{(int)}\left(5/3 + 2/\pi^2\right) + 4f_{12}\left(1 + 2/\pi^2\right)\right]. \tag{30}$$

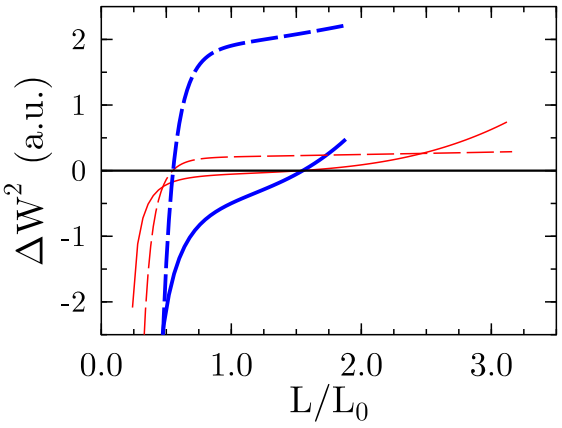

FIG. 3: (Color online) The field dependent part of the variance, $\Delta W^2 = \Omega\gamma_2 F^2$, as a function of the QW width for alloy disorder (solid curves) and identical ($f_{11} = f_{22}$) interface roughness (dashed curves) for two values of the electric field: $F/F_0 = 0.5$ (thinner curves) and $F/F_0 = 1.5$ (thicker curves). Curves are drawn only for the gray shaded area of Fig. 1.

Different signs corresponding to the opposite limits mean that at some particular QW widths, factors $\gamma_2(L_{cr})$ vanish. Numerical analysis shows (see Fig. 3) that these "critical" widths are different for alloy disorder and interface roughness contributions. For parameters used in constructing Fig. 1, L_{cr} for alloy disorder corresponds to $L/L_0 \approx 1.5$, and for interface disorder to $L/L_0 \approx 0.55$. This means that we can effectively turn off the quadratic contribution from one of two sources of the inhomogeneous broadening by growing QW with a width close to the respective critical value, L_{cr}. In this case, all the field induced changes in the exciton broadening will be caused mostly by the other broadening mechanism. In principle, this can allow for unambiguous discrimination between alloy and interface disorder contributions to the exciton line width. For example, growing QW with a size $L/L_0 \approx 1.5$ and measuring field-induced changes in the exciton line-width, we can guarantee that that these changes originate from the interface roughness mechanism only, since contribution from the alloy disorder mechanism vanishes up to the third order in field terms (see the dotted-dashed line in Fig. 4). Different signs of γ_2 for shallow and deep QWs can be explained

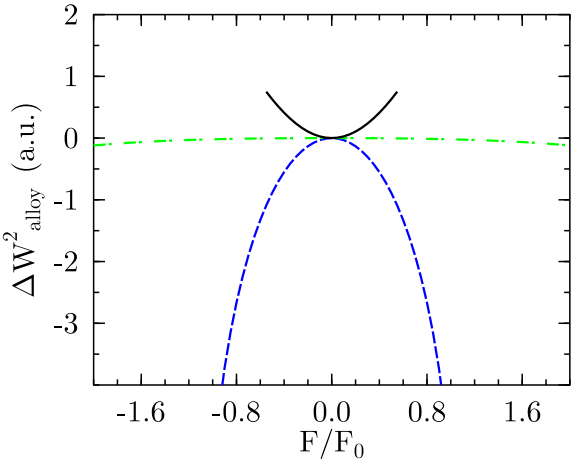

FIG. 4: (Color online) The electric field induced change of the exciton variance ΔW^2_{alloy} for the alloy disorder contribution for three different QW widths: $L/L_0 = 3$ (solid line), $L/L_0 = 1.5$ (dotted-dashed line) and $L/L_0 = 0.3$ (dashed line). Curves are drawn only for the gray shaded area in Fig. 1. Contribution is negative for small thicknesses, and positive at larger thicknesses. The critical value of QW width is determined as a moment when the contribution changes its sign: $L_{cr}^{alloy} \approx 1.5L_0$.

by a competition of two processes. On the one hand, the electric field pushes part of the wave function outside of the well and away from the influences of the disorders, promoting narrowing of the exciton line. On the other hand, the field changes a shape of the wave function, pushing it towards an interface and slightly squeezing. The latter results in greater localization of the wave function and hence, broadens the exciton line. It is clear that the first process dominates for shallow QWs, while the second one prevails for QWs with larger widths.

To estimate quantitatively the critical QW widths we chose the example of $In_{0.18}Ga_{0.82}As/GaAs$. For this QW the material parameters are: $m_e^* = 0.052m_0$, $m_h^* = 0.31m_0$, $\mu_X^* = 0.045m_0$, $U_h = 79\,\text{meV}$, $E_B = 6.7\,\text{meV}$, $a_B = 16\,\text{nm}$. We obtained $L_{cr}^{int} = 1.5\,\text{nm}$ and $L_{cr}^{alloy} = 4.2\,\text{nm}$ for critical interface and alloy disorder lengths respectively. Quantitative values for QWs made from different materials can be readily recalculated using dimensionless values of field and length in Figs. 1-4 and the following formulas for electric field and QW lengths in standard units:

$$\begin{aligned} L[\text{nm}] &= a_B[\text{nm}]\frac{L}{L_0}\sqrt{\frac{\pi^2\mu^* E_B}{2m_h^* V_0^h}} \\ F[\text{V/cm}] &= 10^6 F_0 \frac{E_B[\text{meV}]}{a_B[\text{nm}]}\frac{\mu^*}{2m_h^*}\frac{F}{F_0}, \end{aligned} \tag{31}$$

where F_0 can be extracted from Eq. (16).

In conclusion, we considered effects of an electric field on the inhomogeneous line width of QW excitons. It is shown that interface roughness can result in linear with respect to the field contribution to the exciton line width. This gives rise to the effect of switching between narrowing and broadening of the exciton line by changing field polarity. Quadratic contributions to the field dependence of the line width from both alloy and interface disorders are negative for shallow QWs, but change signs with the increase of the QW depth. These effects reveal a rich physics of inhomogeneously broadened excitons in an electric field, and can be used in applications for narrowing exciton spectral lines and experimental study of the roles of different mechanisms of inhomogeneous broadening of excitons. We would like to stress that even though we considered a simplified model of QW, the incorporation of such effects as valence-band mixing, non-parabolicity of the conduction band, dielectric constant and effective mass mismatches would not change the qualitative results of the paper, which are related to the presence of two competing mechanisms affecting the exciton effective potential rather than to any particular model of the electron (hole) band structure or the confinement potential.

We are grateful to S. Schwarz for reading and commenting on the manuscript. We thank A. Efros for helpful discussions. The work is supported by AFOSR grant F49620-02-1-0305 and PSC-CUNY grants.

* Electronic address: ilya@bloch.nrl.navy.mil

[1] D. A. B. Miller, *et. al.,* Phys. Rev. B **32**, 1043 (1985).
[2] K.K. Bajaj, Mat. Sci. & Eng. R **34**, 59 (2001).
[3] A. L. Efros and M. E. Raikh, in *Optical Properties of Mixed Crystals,* eds. R. J. Elliott and I. P. Ipatova, p. 133 (North-Holland, Amsterdam, 1988).
[4] E. Runge, in *Solid State Physics,* eds. H. Ehrenreich and F. Spaepen (Academic, San Diego, 2002).
[5] M. Herman, D. Bimberg, and J. Christen, Journal of Applied Physics **70**, R1 (1991).
[6] S. Baranovskii and A. Efros, Sov. Phys. Semicond. **12**, 1328 (1978).
[7] R. Zimmermann, Phys. Stat. Sol. B **173**, 129 (1992).
[8] A. Efros, C. Wetzel, and J. Worlock, Phys. Rev. B **52**, 8384 (1995).
[9] C. Weisbuch, R. Dingle, A. Gossard, and W. Wiegmann, Solid State Communications **38**, 709 (1981).
[10] J. Singh, K. K. Bajaj, and S. Chaudhri, Appl. Phys. Lett **49**, 805 (1984).
[11] V. Srinivas, J. Hryniewicz, Y. J. Chen, and C. E. C. Wood, Phys. Rev. B **46**, 10193 (1992).
[12] R. Zimmermann, F. Grosse, and E. Runge, Pure & Appl. Chem. **69**, 1179 (1997); H. Castella and J. W. Wilkins, Phys. Rev. B **58**, 16186 (1998).
[13] I. V. Ponomarev, L. I. Deych, and A. A. Lisyansky, Appl. Phys. Lett **85**, 2496 (2004).
[14] A. E. Meyerovich and A. Stepaniants, Phys. Rev. B **58**, 13242 (1998).
[15] A. E. Meyerovich and A. Stepaniants, Phys. Rev. B **60**, 9129 (1999).
[16] A. E. Meyerovich and I. V. Ponomarev, Phys. Rev. B **65**, 155413 (2002).
[17] I. V. Ponomarev, L. I. Deych, and A. A. Lisyansky, Phys. Rev. B **71**, 155303 (2005).
[18] L. Deych, I. Ponomarev, Phys. Rev. B **71**, 035342 (2005).
[19] G. Bastard, E. Mendez, L. Chang, and L. Esaki, Phys. Rev. B **28**, 3241 (1983);S. Nojima, Phys. Rev. B **37**, 9087 (1988).